Geometric-oriented Signal Processing Lab (GoSPL)

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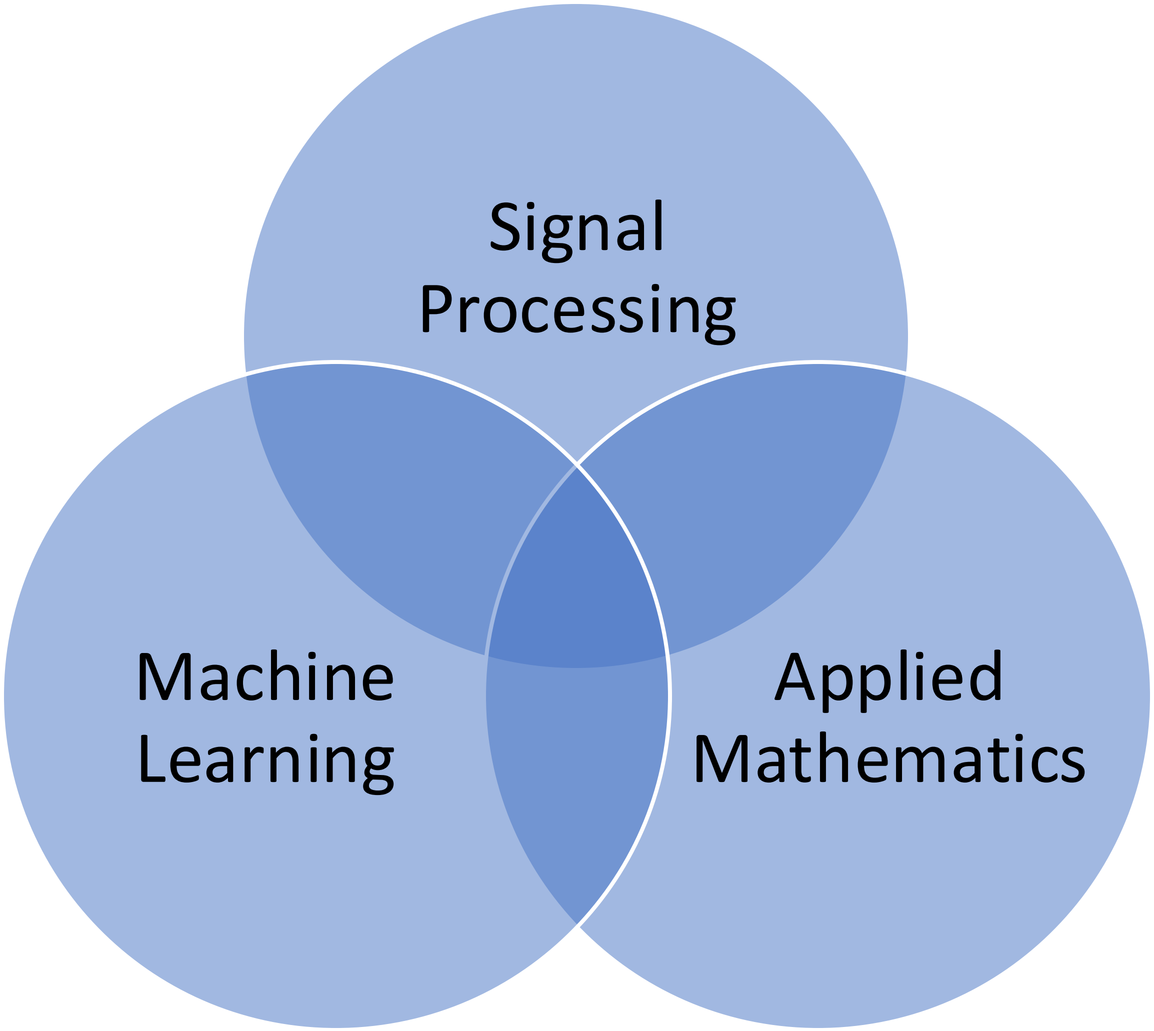
SIPL 50 Years Event

May 2025

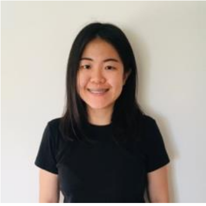
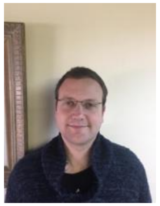
Geometric Signal and Data Processing



Group Members

- PhD students:
  - Ya-Wei Lin
  - Ido Cohen
  - David Cohen
  - Yehonatan Segman
  - Harel Mendelman
  - Gal Maman



- MSc students:
  - Bar Weiss
  - Adi Arbel
  - Or Cohen
  - Yuval Sitton
  - Yoav Harris
  - Hen Ziv
  - Yuval Marsh-Damti
  - Daniel Hassid





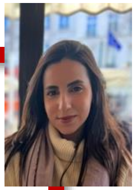


Manifold Learning

Multi-Manifold Learning

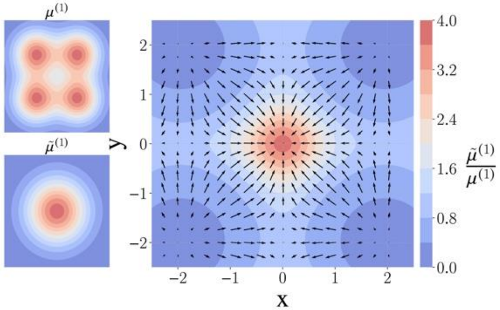
Extensions:

- Feature Selection (ICML’23)
- Supervised and Semi-supervised Manifold Learning (ICLR’25)
- Graph Rewiring for GNNs



Anisotropic Diffusion

- Unpaired Multiview Data
- New cost for OT

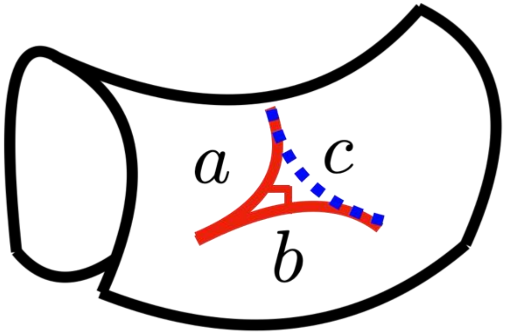


Studying Different Aspects of Geometry

1

Learn the geometry of data

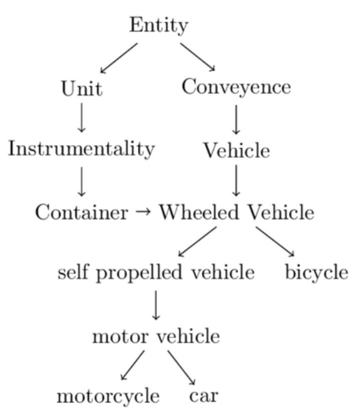
2



Exploit non-Euclidean geometry
of spaces and representations

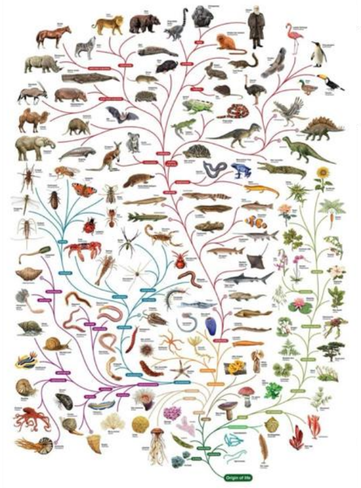
Hierarchical Data Representation in Hyperbolic Spaces

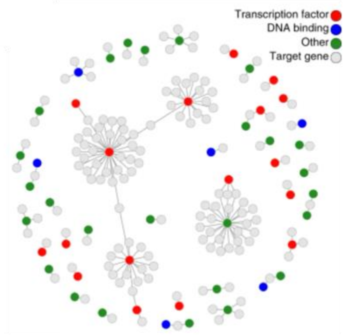
- Hierarchical data is prevalent in many fields



PEmbedding [Nickel et al.]

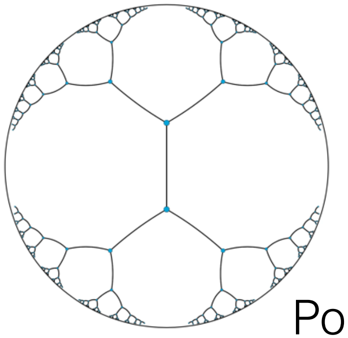
PMFBrain[Gao et al.]


Ontology [Euzenat et al.]

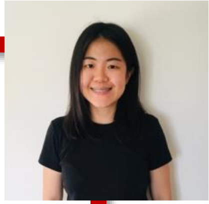


PMSca [Klimovskaia et al.]

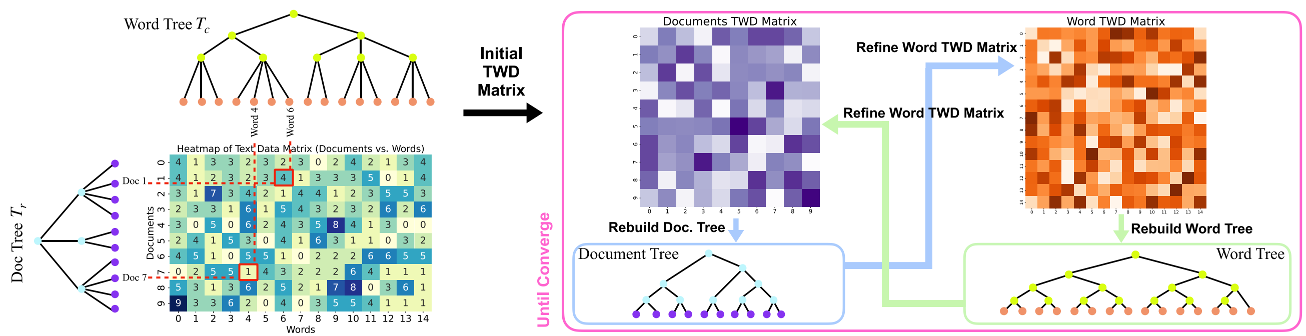
- Why hyperbolic spaces?

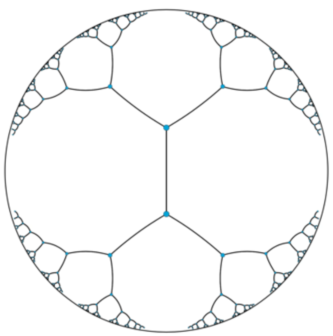

Poincare Disk

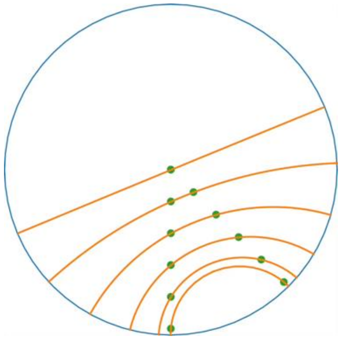
Hierarchical Data Representation in Hyperbolic Spaces



- Hyperbolic Embedding (ICML'23)
- Tree-Learning and Tree Wasserstein Distance (ICLR'25)
- Joint Hierarchical Representation of Samples and Features







Undergraduate SIPL project

- Elad Lavi and Amir Bourvine
- ICASSP'25

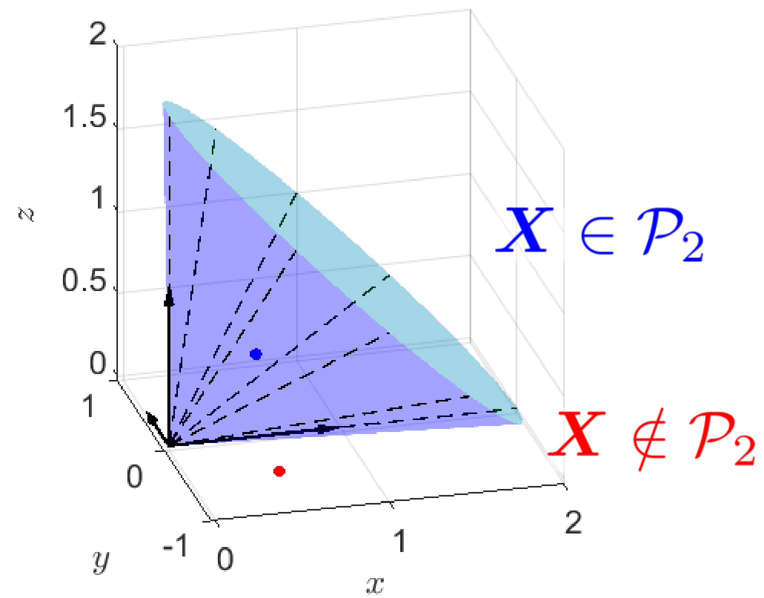
Riemannian Geometry of SPD Matrices

$$\boxed{\boldsymbol{X} \succ 0} \iff \boxed{\forall i : \lambda_i(\boldsymbol{X}) > 0} \iff \boxed{\forall \boldsymbol{v} \neq 0 : \boldsymbol{v}^T \boldsymbol{X} \boldsymbol{v} > 0}$$

$$\boldsymbol{X} = \begin{bmatrix} x & y \\ y & z \end{bmatrix}$$

$$\boldsymbol{X} \succ 0 \iff \begin{cases} x > 0 \\ z > 0 \\ y^2 < xz \end{cases}$$

$$\mathcal{P}_2 = \left\{ \boldsymbol{X} \in \mathbb{R}^{2\times 2} \,:\, \boldsymbol{X} \succ 0 \right\}$$



Riemannian Geometry of SPD Matrices in Array Signal Processing

- <u>Our premise</u>:
  - The spatial information is conveyed by spatial covariance matrices
  - **Covariance matrices are SPD**

- Direction-of-Arrival (DoA) estimation as **covariance matching**:

$$\hat{\theta}_{\mathrm{DS}} = \operatorname{argmin}_{\theta} \left\| \hat{\Gamma} - \Gamma(\theta) \right\|_{F}$$

$$\hat{\theta}_{\mathrm{MVDR}} = \operatorname{argmin}_{\theta} \left\| \hat{\Gamma}^{-1} - \Gamma^{-1}(\theta) \right\|_{F}$$

$$\hat{\theta}_{\mathrm{CM}} = \operatorname{argmin}_{\theta} \left\| W^{H} \left(\hat{\Gamma} - \Gamma(\theta) \right) W \right\|_{\mathbf{F}}$$

$$\hat{\theta}_{\mathrm{COMET}} = \operatorname{argmin}_{\theta} \left\| \hat{\Gamma}^{-\frac{1}{2}} \left(\hat{\Gamma} - \Gamma(\theta) \right) \hat{\Gamma}^{-\frac{1}{2}} \right\|_{F}$$

$$\hat{\theta}_{\mathrm{SPICE}} = \operatorname{argmin}_{\theta} \left\| \Gamma^{-\frac{1}{2}}(\theta) \left(\hat{\Gamma} - \Gamma(\theta) \right) \hat{\Gamma}^{-\frac{1}{2}} \right\|_{F}$$

Riemannian Geometry of SPD Matrices in Array Signal Processing

- <u>Applications</u>:
  - Localization (ICASSP'24)
  - Signal enhancement (ICASSP'25)
  - Interference suppression (TSP'24)
  - Radar and acoustic signals
  - Narrow- and broad-band signals

SIPL

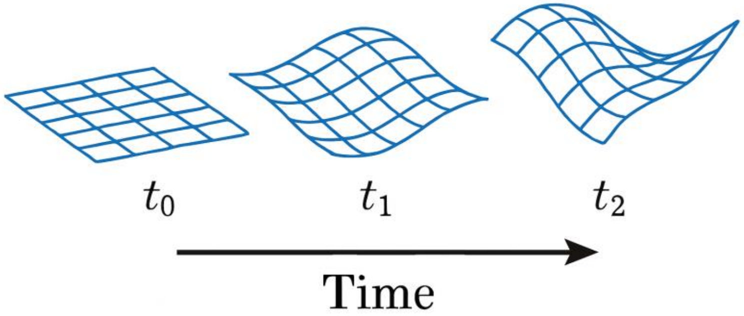
Undergraduate SIPL project
- Or Ronai and Yuval Sitton
- ICASSP'25

Sparse Spectrum Estimation

Manifold Learning

Dynamical Systems

- Spatiotermporal Analysis
- Inference from observations



Sparse Spectrum Estimation

Dynamic Mode Decomposition (DMD) [Schmid, 2010]

↕

Matrix Pencil Method [Hua & Sarkar, 1988]

Detection and estimation of a finite sum of complex exponentials:

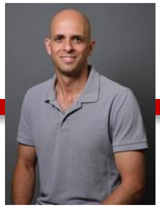
$$y(n) = x(n) + w(n)$$

$$x(n) = \sum_{i=1}^{M} b_i e^{(-\alpha_i + j\theta_i)n}$$

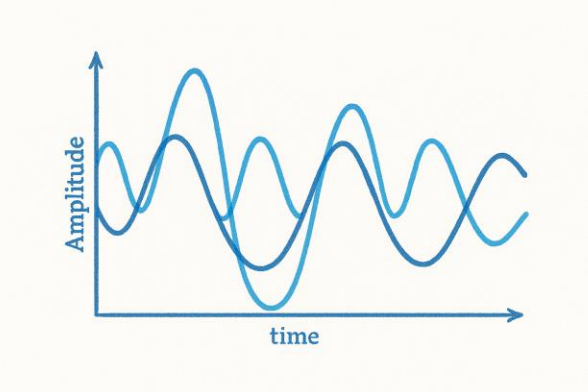
Current challenges:

- Existing MP methods lack theoretical guarantees under noise
- Detection is based on heuristics

Sparse Spectrum Estimation



- Extending MP theory in the presence of noise
- New analysis of the structure of the eigenvectors
- New detection algorithm using the **eigenvectors** rather than the eigenvalues



Sparse Spectrum Estimation

Extensions to related methods:

- Prony's method
- **Matrix Pencil Method**
- MUSIC
- ESPRIT


Applications:

- Radar signals
- Imaging (MRI)
- Super-resolution time-frequency analysis
- Dynamic mode decomposition (DMD)


Thank you